hydrostatic curve based on measurements to 98 kbar is shown for comparison (43). The curves labelled 3rd, 4th, are fits based on low pressure acoustic measurements and finite elastic strain theory (Section 2.33).

These plots show clearly the important features of the compression, namely:

(1) extremely high amplitude elastic waves, up to 150 kbar in Z-cut quartz

(2) loss of rigidity above the elastic limit, as shown by the agreement of the higher pressure shock data with extrapolation of the hydrostatic data

(3) lack of a unique value for the Hugoniot elastic limit.

This behavior implies that yielding is not due to dislocation motion as in a metal, but is analogous (or identical) to fracture. It is shown below that the shear stresses behind the elastic shocks approach the theoretical shear strength of the crystal lattice.

The range of the present data is not sufficient to show clearly the transformation to stishovite, as indicated by Wackerle's higher pressure measurements.

2.33 Finite Strain Theory

Because the strains behind the elastic shocks are relatively large, it is of interest to examine the agreement of the data with predictions of finite strain theory. Predictions are made possible by the work of THURSTON (40) and McSKIMIN (39) and their co-workers on the third-order elastic constants of quartz. Such comparisons should indicate the extent to which third-order constants are sufficient to describe the stress-strain behavior at strains of the order of 5 - 10%. The constants are determined from precise acoustic measurements at strains of less than 0.1%. ANDERSON (44) has already shown that the second and third-order constants alone give reasonably good predictions for hydrostatic compressions of up to about 15% in quartz, provided the constants are used in the Murnaghan logarithmic equation or the Birch equation of state.

Discrepancies between the observed and predicted stress-strain curves can be used alternatively to evaluate fourth and higher order constants, or to examine the effects of adopting alternate definitions of strain, as suggested by KNOPOFF (45). Finally, to the extent that the third-order constants give adequate predictions, the stresses tangential to the shock fronts can be calculated from the observed stresses normal to the fronts and, hence, the shear stresses sustained (momentarily) by the crystal can be deduced.

A. Finite Strain Fundamentals*

Denote the coordinates of a mass element in an initial (unstrained) coordinate system by a_i, and the coordinates in a final (strained) system by x_i, with the transformation given by,

$$x_i = x_i(t, a_1, a_2, a_3),\ i = 1, 2, 3 \qquad (2.13)$$

where

$$a_i = x_i(t_0, a_1, a_2, a_3)$$

and t_0 is a reference time. The x_i are thus Eulerian, or spatial, coordinates and the a_i Lagrangian, or material, coordinates.

For this transformation one can derive an expression for the ratio of specific volumes:

$$V/V_0 = J = \frac{\partial x_i}{\partial a_s} \qquad (2.14)$$

*This section is a summary of portions of the theory as presented by THURSTON (46).